

KINEMATIC SUPPORT USING ELASTIC ELEMENTS

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ABSTRACT

In this paper we review the design of kinematic supports using elastic elements. The two standard methods (cone, Vee and flat and three Vees) are presented and a design example involving a machine tool metrology bench is given. Design goals included thousandfold strain attenuation in the bench relative to the base when the base strains due to temperature variations and shifting loads. Space applications are also considered.

INTRODUCTION

The application of kinematic principles to the support of devices is quite old. Evans [1987] traces its evolution to the eighteenth century. Maxwell (Niven [1890]) gives the clearest early statement of principles and is often quoted. The idea is to constrain only as many degrees of freedom as are required by the application but no more. For example, a slideway that must move in one direction only should be constrained by exactly five points, no more or no less. If less, there are extra degrees of freedom and the motion will not be rectilinear. If there are more constraints, there can be stresses built up between the support points, which results in strain in the supported body. In this discussion, we are considering the mounting of strain-sensitive bodies such as optical benches, inertial systems with several components that must have accurate relative orientation, precision measuring systems such as a metrology bench, etc. These bodies must be supported. Six degrees of freedom must be fixed, but if the base upon which they are mounted is strained, it must not be propagated into the device.

There are two classical configurations for accomplishing this (See Fig. 1a and b):

- cone Vee and flat
- three Vee's

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In Fig. 1a, the left corner is fixed by the contacts of its mount in three degrees of freedom of translation. However, with only this support, the device can be rotated about any of the three axes. The right hand forward mount rests in a Vee which provides two more constraints, but allows rotation about the line joining the two first mounts. The final constraint is supplied by a flat upon which the third corner rests. Similarly, for the three Vees, there are two constraints at each of the corners. In both cases, six degrees of freedom are constrained.

On the earth, this is frequently done by letting gravity press three balls against the constraints. The assumptions are that

- the weight places the balls in contact with their constraints and holds them there.
- if strain occurs, the balls are free to slide along the surfaces on which they rest.

Thus, the strain of the base is not propagated into the supported device.

While these mounts provide a high degree of repeatability if the device is removed and returned, the assumptions are not always satisfied. In space, there is no weight to ensure that the constraints are active. It is necessary to provide a force pushing the balls against the hard points. Since this process can cause strain in the body, care is required to keep the constraining force constant. Furthermore, if the base strains, the surfaces are not frictionless and a small and unfortunately unpredictable amount of strain gets propagated to the body. Both of these disadvantages can be overcome by employing elastic elements in supporting the device. Proper scaling permits adequate stiffness for support while providing acceptable compliance for the attenuation of strain induced in the base.

ELASTIC ELEMENTS

The most common form of elastic element is the flexure which is a flat sheet of material (see f. ex. Jones[1961], Jones[1962], Andreeva[1962], Geary[1954]). For example, the Bendix crossed-flex hinges use these plates in bending. Each plate has three degrees of freedom that are stiff. In Fig. 2a, one can see by inspection that the flexible degrees of freedom are in translation along Z and rotation about the X and Y axes. Combinations of these plate flexures can be used to increase or decrease the number of degrees of freedom. For example, the figure shown in 2b is flexible only in translation along Z, whereas in Fig. 2c, the combination shown provides stiffness only in the X direction.

A more direct way of providing stiffness in one degree of freedom with compliance in the other five is to use a rod. Rods can be combined easily to constrain additional degrees of freedom by mounting them perpendicular to each other at a single location. When the rods are piston actuators, six degrees of freedom can be

controlled as is done in some moving base simulators for pilot training and research (Stewart[1965], Dagalak[1987]).

In a satellite project at Stanford, supported by sub-contract to Lockheed Missiles and Space Company, we intend to mount a solar array with six rods of this type to avoid over-constraints that could produce unnecessary stresses on the solar arrays or the vehicle (see Fig. 3). We are also involved in some precision manufacturing work in which a machine tool is being developed that requires a separate metrology bench which will be free from the strains in the machine due to temperature variations, changing locations of weight and machining forces. In the latter example, we have made individual supporting elements which are the kinematic equivalent of three elastic V's.

Thus by starting with a requirement for adequate strength and some minimum level of stiffness, one can establish the necessary size and slenderness ratio which will provide an adequate mounting employing elastic elements. Depending upon the application, one may choose to use three V's or a cone V and flat configuration, as shown in Figure 4. If there is one point in the body whose position has special significance with respect to the base, then a cone, V and flat may be the most appropriate. The three rods of the cone should be placed and oriented so that they pass through the critical point, thus minimizing any translation of this point due to rotation of the body relative to the base.

The problem of damping the motions of the supports has not been solved satisfactorily. Jim Bryan has suggested using wire cable for the rods to allow internal friction in applications where slight stick slip within the cable would be an acceptable disturbance. This would also allow much shorter cables than rods of equal strength due to the cable's softness in bending.

DESIGN FOR PERFORMANCE

Kinematic mounting of a body on a base poses two kinds of questions regarding performance:

- How well does each support constrain its assigned degrees of freedom.
- How rigidly do all supports collectively mount the body on the base.

The first class of questions arises from our need to simulate the motion of a ball resting in a conical hole with three sides, in a V-shaped groove with two sides or on a flat surface in all cases without any friction. When using flexible elements in order to eliminate sliding contact in hinges or bearings, some stiffness in unconstrained degrees of freedom is unavoidable. However, this is often better than the erratic stick-slip motion of two surfaces. In the following we shall concentrate on pairs of flexible perpendicular rods used as depicted in Fig. 5 for kinematic mounting.

Each pair simulates a ball in a V-shaped groove. Defining a coordinate system and symbols as in Fig. 5 we obtain the following stiffness matrix:

$$K = \begin{pmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{24EI}{L^3} & 0 & 0 & 0 & -\frac{12EI}{\sqrt{2}L^2} \\ 0 & 0 & \frac{EA}{L} & 0 & -\frac{REA}{L} & 0 \\ 0 & 0 & 0 & \frac{EI}{L}(\frac{24R^2}{L^2} + 6.79) & 0 & 0 \\ 0 & 0 & -\frac{REA}{L} & 0 & \frac{E}{L}(8I + AR^2) & 0 \\ 0 & -\frac{12EI}{\sqrt{2}L^2} & 0 & 0 & 0 & \frac{4.8EI}{L} \end{pmatrix}$$

which relates forces and torques to displacements through the equation $f = Kd$ where

$$f = [f_x, f_y, f_z, M_x, M_y, M_z]^T$$

$$d = [d_x, d_y, d_z, \theta_x, \theta_y, \theta_z]^T$$

The above stiffness matrix reflects our choice to mount each rod at 45 degrees from the vertical to provide equal stiffness in x and z directions. In the stiffness matrix, we want elements corresponding to F_x/d_x and F_z/d_z to be as large as possible while keeping all other entries close to zero to attenuate the propagation of strain in the base to the body. Examination of the entries leads to the intuitive conclusion that we should keep the rods long (large L) and thin (small D) and they should be attached as close as possible to their point of intersection (small R). We can look at the ratio between F_x/d_x and F_y/d_y as a measure of quality and notice that this ratio is only proportional to the slenderness ratio (L/D) squared with no dependence on material properties. If we make the rods very long and thin, we either compromise their ability to constrain the degrees of freedom that the support is designed to eliminate or they get excessively long. In the stiffness matrix, this appears as a decrease in the F_x/d_x and F_z/d_z stiffnesses although the ratio between F_x/d_x and F_y/d_y increases (unless we keep D^2/L constant as we increase L). This ratio, L/D , is therefore not a sufficient measure of the quality of the design, which leads to the second class of questions, namely those of collective performance of all three supports.

For each application there are normally several criteria that the design must meet. We may have to consider

- adequate strength of the mount
- overall stiffness of the body's mounting on the base
- natural frequencies and modeshapes of the kinematically mounted body
- strain propagation from base to body

The issue of strain propagation is addressed both by the design of each support, their location and orientation and the stiffness of the body. Since there is some stiffness

in unconstrained degrees of freedom, there will always be some strain introduced in the body when the base strains. By carefully laying out the location and orientation of the supports it is possible to influence the nature of the strains generated in the body. We can for example arrange the supports so that for a given strain in the base, minimum strain energy is accumulated in the body or arrange them so that angular distortion of the body (when, say, the base expands uniformly due to increase in temperature) is minimum.

Although analysis of the problem must reflect each application, a simple example is in order. Let us assume that the body to be suspended is an isosceles triangular frame with supports, characterized by the above stiffness matrix, mounted at its corners. Referring to Fig. 6, a relationship is sought between the displacement of the supports at the base (z_3 and z_4) and the resulting displacement of the corners of the triangle (z_1 and z_2).

Let angles ϕ and θ , frame stiffnesses s_1 and s_2 , support stiffness (i.e. F_y/d_y) k , and lengths L_1 and L_2 be as defined in Fig. 6. The following relationships then hold for the geometry of the frame as it strains:

$$\delta L_1 = \cos(\phi)z_1 + \cos(\theta - \phi)z_2$$

$$\delta L_2 = 2z_2 \sin(\theta)$$

The total potential energy resulting from strains may then be written in terms of displacements:

$$E = s_1(\delta L_1(z_1, z_2))^2 + \frac{1}{2}s_2(\delta L_2(z_1, z_2))^2 + \frac{1}{2}k(z_3 - z_1)^2 + k(z_4 - z_2)^2$$

Now differentiating this with respect to z_1 and z_2 and setting the result to zero provides a relationship between frame displacements and base displacements:

$$\begin{pmatrix} 2(\frac{s_1}{k})\cos^2(\phi) + 1 & 2(\frac{s_1}{k})\cos(\theta - \phi)\cos(\phi) \\ 2(\frac{s_1}{k})\cos(\theta - \phi)\cos(\phi) & 2(\frac{s_2}{k})\cos^2(\theta - \phi) + 4(\frac{s_2}{k})\sin^2(\theta) + 2 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} z_3 \\ 2z_4 \end{pmatrix}$$

One might now, for example, set the z_3 and z_4 values as proportional to the distance of the corresponding corners to the point where lines of unconstrained translation for each support intersect. Again referring to Fig. 6 this corresponds to uniform strain (ϵ) in the base resulting in $z_3 = \epsilon R_1$ and $z_4 = \epsilon R_2$. Now if no angular distortion is required, the angle θ would have to be adjusted until the solution z_1 and z_2 is also proportional to R_1 and R_2 . If minimal strain in the frame is the object, the expression for the total potential energy can be minimized by adjusting θ . Our calculations have led us to the following conclusions:

- If all three members of the frame are equally stiff, both minimum strain energy and no angular distortion of the frame are achieved by letting the lines of free translation intersect in the centroid of the triangle.
- If the compliance of the members of the triangle is proportional to their length (as in the case of straight, homogeneous beams of uniform cross section), minimal strain and no angular distortion require different angles θ . However, strain energy in the no distortion case is insignificantly higher than in the minimal strain case (order of 1%).

As pointed out earlier, these considerations must only go as far as allowed by requirements for overall stiffness and natural frequencies.

For the more complex case of nonsymmetric geometry, the analysis proceeds in a similar fashion, only with more symbols to manipulate. It is interesting to note that if the lines of unconstrained translation do not all intersect in one point, the frame may rotate as well as undergoing distortion.

A MACHINE TOOL METROLOGY BENCH

The motivation for our study comes from the field of precision machine tools. In our laboratory at Stanford, we are building an ultra precision turning machine to be capable of turning parts with surfaces of optical quality and repeatability on the order of microinches. This machine is composed of a hydraulic laminar flow spindle, a hydraulically controlled cross-slide on hydrostatic bearings and a high bandwidth tool actuator. All these parts along with the granite slab on which they are mounted are showered with tightly temperature controlled oil to prevent dimensional changes due to temperature fluctuations.

To achieve such high precision in machining it is necessary to employ sophisticated metrology which in our case is based on laser interferometry. All metrology equipment is mounted on a separate metrology bench of granite which surrounds the machinery (Bryan[1979]). Since the geometry of this metrology bench is the standard to which all measurements of the machinery is referred, we require that any strains that may arise in the foundation of the machine should not propagate into the metrology bench while at the same time it is rigidly fixed to the foundation. These requirements are met by kinematic mounting. Since no particular point on the metrology bench is more significant than others, we chose to use the three V configuration, this giving the advantage of identical design of all three supports.

The metrology bench weighs 680 kg (1500 lb), it is shaped like the letter C and made of 0.254 m (10 in.) thick granite. Primary considerations in design of its supports were:

- Sufficient strength to carry its weight.

- The weight carried in tension to avoid buckling.
- High stiffness in constrained directions for natural frequencies greater than 40 Hz.
- Low stiffness in free directions to attenuate strain by 10^3 .
- Supports located symmetrically about the mass center for equal loading.
- No rubbing contact.
- Stops to prevent yielding of flexible parts.
- Easy manufacture.

Each support must be functionally equivalent to two rods in a V arrangement (see Fig. 3b). Due to the heavy weight of the bench, it is best to load the rods in tension rather than compression to prevent buckling. In our design this is done by suspending the rods from a column rising from the base with the bench attached to the lower end of the rods. They are set at an angle of 45 degrees from the vertical to make the supports equally stiff in all directions in the plane of the rods. It is important that the first natural frequency of the bench, when moving as a rigid body on all three supports, be sufficiently high that the mass attenuates greatly the associated motion. The orientation that was chosen for the supports (see Fig. 7) reflects this and a desire to make the collective stiffness of the supports equal, i.e. an external horizontal force acting on the bench would cause equal deflection independent of direction. As admitted earlier, no good way has been found to passively damp motions of the supports without compromising other necessary qualities.

Design for fabrication involved several considerations:

- Monolithic or assembled
- Methods of assembly: brazing or bolting
- Material suitable for strength, machining and brazing
- Cost of material and fabrication
- Corrosion resistance

Making each support truly monolithic is difficult in fabrication and since no disassembly is needed after fabrication, it was decided to make each support of five pieces and braze them together: footplate, column, carrier beam and two rods (see Fig. 8). Being easy to machine and corrosion resistant, stainless steel alloy 303 was chosen for all pieces other than rods. Drillrod was chosen as the material for the rods for its strength, cost and availability. Minimum diameter to support the load was found to be 6.35 mm (1/4 in.) while the length could be no greater than 0.119 m (4.7 in.) due to space constraints. This results in a stiffness ratio for constrained and free directions of 234 and an overall strain attenuation from base to bench of 1000.

The first modeshape is shown in Fig. 9. We see that two of the supports are moving in their flexible direction. The associated natural frequency is 46 Hz. The next five natural modes, the highest frequency of which is 80 Hz, also principally

involve the supports, with the bench nearly rigid. The seventh mode is the first one to include significant strain of the bench and occurs at 200 Hz.

CONCLUSIONS

Kinematic support using elastic elements offers some advantages. In a weightless environment it can constrain in tension or compression so external loading is not needed. The strain propagation is systematic and smooth which may be preferred over the stick-slip adjustments of contacting surfaces that must slide. Rods offer a simple design choice with one degree of freedom constrained by each of six rods. Greater lateral and rotational compliance for a given strength can be achieved with stacked flexures however. By selection of location and orientation as well as support design, the strain propagated can be managed in a variety of different ways permitting some selection in design to minimize distortions which may be particularly undesirable.

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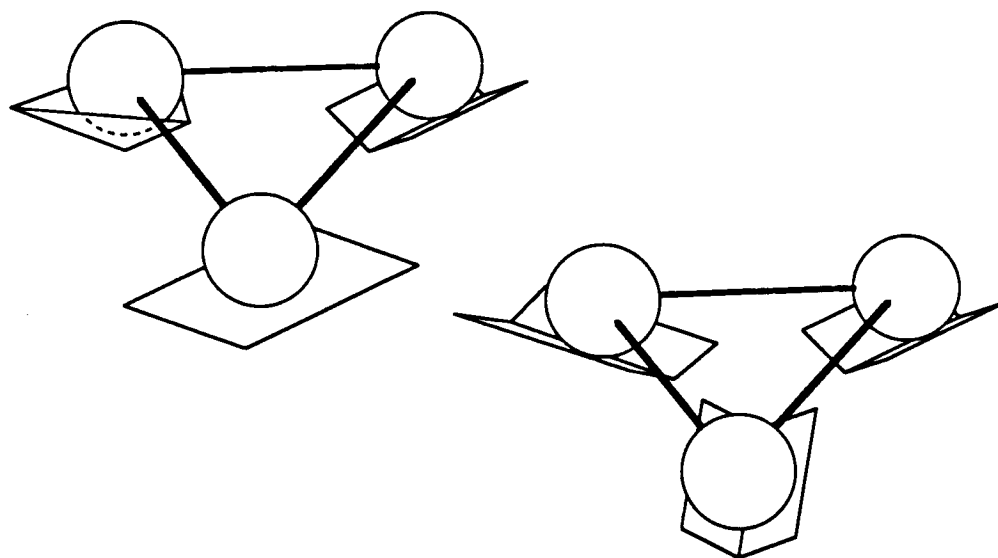


Figure 1: a) Balls resting in conical hole, in V-shaped groove and on a flat surface. b) All three balls in V-grooves.

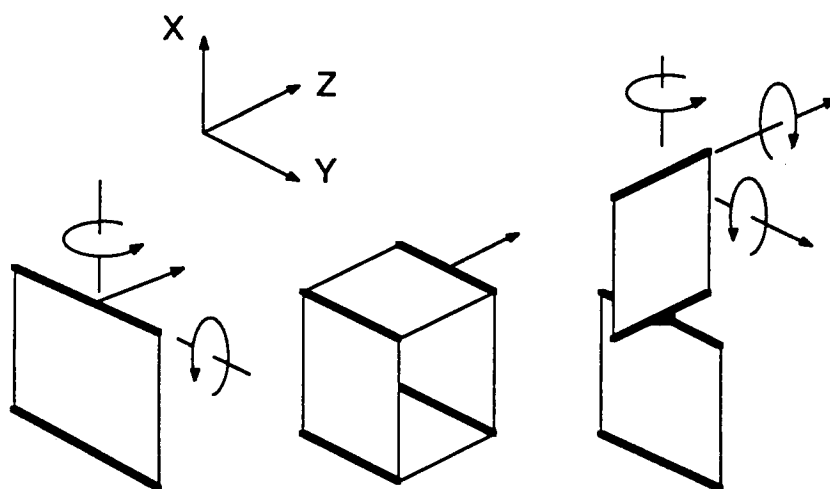


Figure 2: Flexures with a) three, b) one and c) five compliant degrees of freedom.

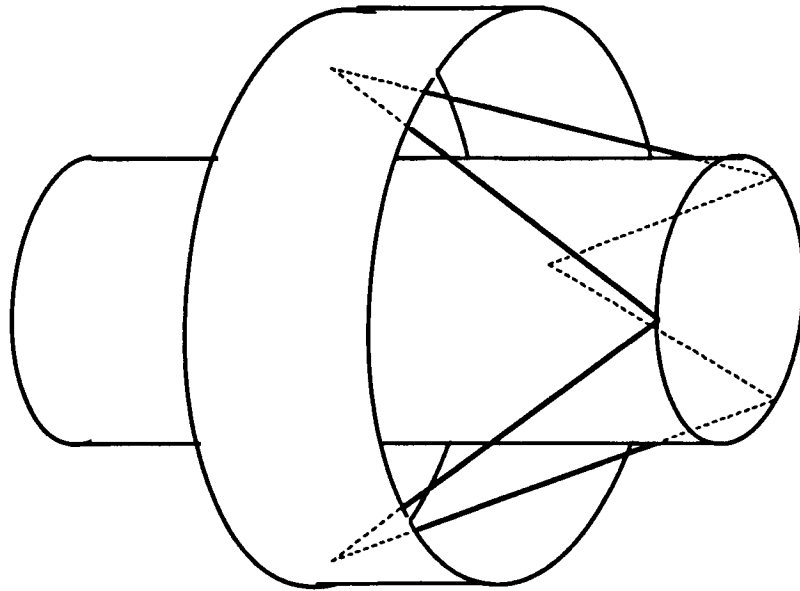


Figure 3: Kinematic suspension of solar array on satellite.

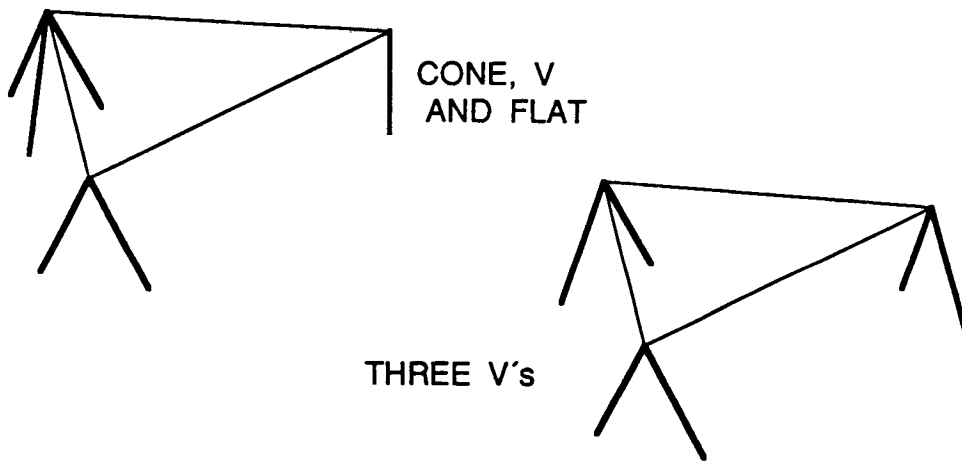


Figure 4: Cone, Vee and flat or three VeEs implemented with flexible rods.

